

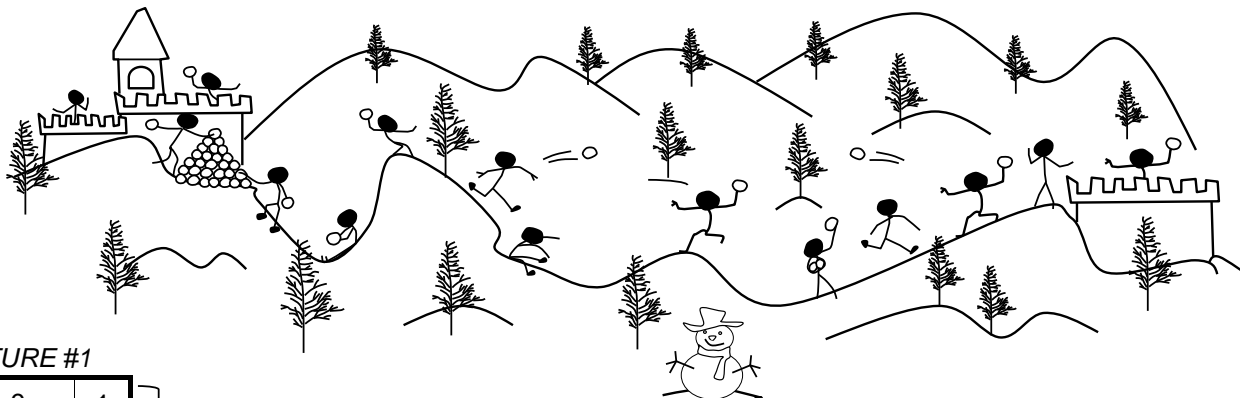
SNOWBALLS - ALGEBRAIC EQUATIONS

NAME: _____

The Sticks challenged the Line Segments to a snowball fight. They each built a fort out of snow to defend. The Sticks decided to be defensive and placed all 40 teammates around their fort. They were arranged in the shape of a square and had 11 guards defending each side of the fort (See *picture #1* below). The Line Segments decided to be a little more offensive and everyone just attacked.

During the first four assaults by the Line Segments, the Sticks lost 4 soldiers each time and sustained heavy damage to their fort, and after the fifth assault they only had 22 people left. Even though they lost soldiers, they always had 11 people defending each side of the fort. How was this possible?

Use the squares to help show how the guards were re-arranged after each of the 5 attacks to still have 11 people on each side of the fort.



PICTURE #1

1	9	1
9	40 GUARDS	9
1	9	1

40 GUARDS TOTAL
AND 11 ON EACH
SIDE OF THE FORT.

Helpful information:

Always 11 on each side.

n = the total number of soldiers still guarding the fort.

c = the number of soldiers in one corner.

m = the number of soldiers between two of the corners.

AFTER 1st ATTACK

c	m	c
m	n GUARDS	m
c	m	c

$n =$

$c =$

$m =$

AFTER 2nd ATTACK

c	m	c
m	n GUARDS	m
c	m	c

$n =$

$c =$

$m =$

AFTER 3rd ATTACK

c	m	c
m	n GUARDS	m
c	m	c

$n =$

$c =$

$m =$

AFTER 4th ATTACK

c	m	c
m	n GUARDS	m
c	m	c

$n =$

$c =$

$m =$

AFTER 5th ATTACK

c	m	c
m	n GUARDS	m
c	m	c

$n =$

$c =$

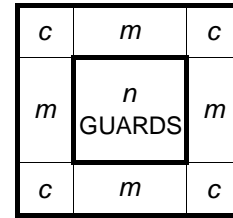
$m =$

Challenge: Write two algebraic equations that can be used to help solve how the soldiers were re-arranged after each attack to still have 11 on each side. Use n , c , and m as the variables. For the 5th attack the equations will be helpful but not perfect, why?

ANSWER AND HELP

The first 4 attacks they lost 4 people each time, so they started with 40, then 36, 32, 28, and 24. After the fifth attack they had 22 people left.

You know they had 11 people on each side of the fort every time.



If you use the square you can actually write two algebraic equations.

The first one should be related to the total number of guards.

$$4m + 4c = n$$

where m equals the number of guards in the middle of two corners, c equals the number of guards in one corner, and n equals the total number of people still guarding the fort.

The other equation should be related to the number of guards on one side of the fort.

$$m + c + c = 11 \text{ or } m + 2c = 11$$

where the variables mean the same as above.

Now you can re-write the second one by moving the $2c$ to the other side ($m + 2c = 11$).

$$m = 11 - 2c$$

and substitute it into the first equation ($4m + 4c = n$). It would look like this:

$$4(11 - 2c) + 4c = 36, m \text{ now equals } 11 - 2c$$

$n = 36$ because they lost 4 people after the first attack ($40 - 4 = 36$).

Solve:

$$4(11 - 2c) + 4c = 36$$

$$44 - 8c + 4c = 36$$

$$44 - 4c = 36$$

$$-4c = -8$$

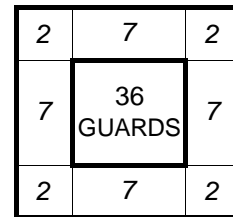
$$c = 2$$

So there are 2 guards in each corner and since $m + 2c = 11$ and $c = 2$ then...

$$m + 2(2) = 11$$

$$m = 11 - 4$$

$$m = 7$$



So after the first attack the fort would look like this:

You can use this to solve the other attacks.

Now if you were struggling with the algebraic equation you could use your pattern / puzzle solving skills and see that every time they lose 4 people c increases by one and m decreases by 2. See below for all answers.

AFTER 1st ATTACK

2	7	2
7	36 GUARDS	7
2	7	2

AFTER 2nd ATTACK

3	5	3
5	32 GUARDS	5
3	5	3

AFTER 3rd ATTACK

4	3	4
3	28 GUARDS	3
4	3	4

AFTER 4th ATTACK

5	1	5
1	24 GUARDS	1
5	1	5

AFTER 5th ATTACK

6	0	5
0	22 GUARDS	0
5	0	6

Last, the reason the equation for the 5th attack is not perfect is that $c = 5.5$.

The equations are: $4m + 4c = 22$ and $m = 11 - 2c$ so,

$$4(11 - 2c) + 4c = 22$$

$$44 - 8c + 4c = 22$$

$$44 - 4c = 22$$

$$-4c = -22$$

$$c = 5.5$$

But you can not have 5.5 people so you must use 5 and 6 for the corners and 0 for the middle.